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LOCAL INSTABILITY OF SYMMETRICAL RECTANGULAR TUBES UNDER AXIAL COMPRESSION

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SUMMARY

A chart is presented for the coefficient in the formula for the critical compressive stress at which cross-sectional distortion begins in a thin-wall tube of rectangular section symmetrical about its two principal axes. The energy method of Timoshenko was used in the theoretical calculations required for the construction of the chart. The deflection equation used in this method was selected to give good accuracy. The exact values given by solution of the differential equation were calculated for a number of cases and it was found that the energy solution was correct to within a fraction of 1 percent.

The calculation of the critical compressive stress at stresses above the elastic range is also discussed. In order to demonstrate the use of the formulas and the chart in engineering calculations, several illustrative problems are included.

INTRODUCTION

In the design of compression members for aircraft, whether they be stiffeners in stressed-skin structures or struts in trussed structures, the allowable stress for the member is equal to the lowest strength corresponding to any of the possible types of failure. In reference 1, all types of column failure are classed under two headings:

(a) Primary, or general, failure.

(b) Secondary, or local, failure.

Primary, or general, failure of a column is defined as any type of failure in which the cross sections are translated, rotated, or both translated and rotated but not distorted in their own planes (fig. 1). Secondary, or local, failure of a column is defined as any type of failure in which the cross sections are distorted in their own planes but not translated or rotated (fig. 2). Consideration is given in this paper only to local failure.

One of the factors to be considered in a study of local failure is the critical compressive stress at which the cross section begins to distort. This critical stress can usually be given in coefficient form. The purpose of this paper is to present a chart that will be useful in establishing the coefficient to be used in calculating the critical compressive stress at which cross-sectional distortion begins in a thin-wall rectangular tube symmetrical about its two principal axes.

The calculations required to evaluate the coefficient plotted in the chart were made by the energy method of Timoshenko. (See reference 2, p. 324, art. 62.) The exact values of the coefficient given by solution of the differential equation (reference 2, p. 337, art. 65) were also calculated for a number of cases and the energy solution was found to be correct to within a fraction of 1 percent. Because the calculations are long and were made as a part of a more extended study of local failure in thin-metal columns, they have been omitted from this paper.

CHART

The calculation of the critical compressive stress at which cross-sectional distortion begins in a symmetrical rectangular tube is, in reality, a problem in the buckling of thin plates, proper consideration being given to the interaction between adjacent walls of the tube. Timoshenko has given the critical stress for a rectangular plate under edge compression in the following form (reference 3, p. 603):

$$f_{crit} = \frac{k\,\pi^2\,E\,t_h{}^2}{12\,(1-\mu^2)\,h^2} \tag{1}$$

where

E is tension-compression modulus of elasticity for the material.

μ, Poisson's ratio for the material.

t_h, thickness of the plate.

h, width of the plate.

k, a nondimensional coefficient that depends upon the conditions of edge support and the dimensions of the plate.

Equation (1) can be used to calculate the critical compressive stress at which cross-sectional distortion begins in a thin-wall tube symmetrical about its two principal axes. In this case the values of k are obtained from figure 3. The symbols h and t_h are the width and thickness, respectively, of the wider pair of walls; b and t_b refer to the narrower pair of walls. The curves in figure 3 were established by plotting the calculated values of k, given in table I, for the energy solution.

LIMITATIONS OF CHART

The chart of figure 3 must be considered as approximate. For engineering use, however, it may be regarded as a close approximation because the exact values given by solution of the differential equation show that the energy solution is correct to within a fraction of 1 percent. (See table II.)

The values of k given in the chart are the minimum values possible for a tube of infinite length. For engineering use, however, these values will apply to any tube having a length greater than the width of the walls that have the larger ratio of width to thickness. The length of all tubes likely to be encountered in aircraft design will thus fall within the range to which the chart applies. It should be mentioned that, for very short tubes where length does have an appreciable effect, the values of k given in figure 3 are conservative.

The values of k given herein apply to tubes in which the material is both elastic and isotropic. Steel, aluminum alloys, and other metallic materials usually satisfy these conditions, provided that the material is stressed within the elastic range. When a material is stressed above the proportional limit in one direction, it is no longer elastic and is probably no longer isotropic. In a later section of this report it is shown how equation (1) and the chart of figure 3 may be used to calculate the critical stress when the rectangular tube is loaded beyond the proportional limit.

DEFLECTION EQUATION

The previously mentioned deflection equation used in the energy solution had the following form for each wall of the tube:

$$w = \left[\frac{4\ U}{b^2}\ (by - y^2) + B \sin\frac{\pi y}{b}\right] \sin\frac{n\pi x}{L} \tag{2}$$

where

- w is deflection normal to wall.
- L, length of wall equal to length of tube.
- n, number of half waves that form in the length of the wall. The ratio L/n is therefore the half-wave length of a wrinkle in the direction of the length.
- b, width of wall concerned.
- x and y, coordinates measured from end and side of wall, respectively.
- U and B, coefficients. The values of U and B for one pair of opposite walls are expressed in terms of U and B for the other pair of opposite walls by the use of the conditions that the rotation at the edge of adjacent walls be equal and that the bending moments at the edge of adjacent walls be in equilibrium. The ratio of U/B for one wall and L/n are then given values that cause the critical stress to be a minimum.

DISCUSSION OF CHART

When t_b/t_h is equal to or greater than 1, the wider of the opposite walls of the rectangular tube are the weaker. The curves for t_b/t_h of 1 and 2 in figure 3 therefore show how the strength of the wider walls is affected by the width of the adjacent walls. It should be noted that these curves are smooth, having no sharp break such as the curve t_b/t_h for 0.5 has at a b/h value of 0.65.

When t_b/t_h is less than 1, the wider pair of the opposite walls are the weaker provided that b/h is less than a definite value. When b/h is greater than this value, the narrower pair of the opposite walls are the weaker. At the value of b/h where the weaker walls change from the wide to the narrow side of the rectangle, there is a break in the curve for k. For $t_b/t_h = 0.5$, this break comes approximately at $b/h = 0.65$.

CRITICAL STRESS FOR LOADING BEYOND THE PROPORTIONAL LIMIT

In the elastic range, the critical compressive stress for an ordinary column that fails by bending is given by the Euler formula. Beyond the proportional limit, which marks the upper end of the elastic range, the reduced slope of the stress-strain curve requires that an effective modulus $\bar{E}$ be substituted for Young's modulus E in the Euler formula. The value of $\bar{E}$ is sometimes written as τE,

$$\bar{E} = \tau E \qquad (3)$$

The value of τ varies with stress. By the use of the double-modulus theory of column action, theoretical values of τ can be obtained from the compressive stress-strain curve for the material (reference 3, p. 572, art. 37, and references 4 and 5). Tests show that, in practice, theoretical values of τ, derived on the assumption that no deflection occurs until the critical load is reached, are too large. It is therefore best, for practical use, to obtain the values of τ from the accepted column curve for the material in the manner outlined in the illustrative problem. The values of τ thus obtained take into account the effect of imperfections that cause deflection from the beginning of loading as well as other factors that may have a bearing on the strength.

For cross-sectional distortion of a thin-wall rectangular tube, the critical compressive stress in the elastic range is given by equation (1). Above the proportional limit, the critical compressive stress is given by equation (1) with an effective modulus ηE substituted for Young's modulus E, or

$$f_{crit} = \eta \frac{k \pi^2 E t_h^2}{12 (1-\mu^2) h^2} \tag{4}$$

In the absence of adequate test data, the value of η cannot be definitely established. It is reasonable to expect, however, that η and τ are related in some way. On the assumption that η is a function of τ, several possible relations were studied.

When an ordinary column begins to deflect, failure is resisted by the longitudinal bending stiffness of the elemental volumes of material composing the member. The reduced critical strength at stresses beyond the proportional limit is, therefore, explained by a reduction in the longitudinal bending stiffness, which is caused by the smaller slope of the stress-strain curve.

When cross-sectional distortion begins in a thin-wall rectangular tube, failure is resisted by the following characteristics of the elemental volumes of material composing the walls of the tube:

1. Longitudinal bending stiffness.

2. Torsional stiffness.

3. Transverse bending stiffness.

The reduced critical strength for local failure at stresses beyond the proportional limit is, therefore, similarly explained by the varying reductions in 1, 2, and 3 caused by the smaller slope of the stress-strain curve.

In article 71 of reference 2, Timoshenko discusses the effect of certain reductions in 1, 2, and 3 on the critical stress for a simply supported plate under edge compression.

In the following discussion, the general principles of this procedure are used.

The differential equation of the deflection surface of a plate under edge compression in the x direction is

$$f\,t\,\frac{\partial^2 w}{\partial x^2} = -D\left[\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \qquad (5)$$

where

f is stress on the loaded edges of the plate.

t, thickness of the plate.

$D = \dfrac{E\,t^3}{12\,(1-\mu^2)}$, flexural rigidity of the plate.

The left side of equation (5) is concerned with the external forces on the plate that cause buckling, whereas the right side is concerned with the internal resistance of the plate to buckling. The first and the third terms in the brackets on the right side of equation (5) are concerned with the longitudinal and the transverse bending, respectively, whereas the second term is concerned principally with the torsional stiffness.

It is assumed that, when a plate under edge compression is loaded beyond the proportional limit, the three terms in the bracket on the right side of equation (5) are reduced by multiplying each by a different function of τ, where τ is defined by the relation,

$$\tau = \frac{\bar{E}}{E} \qquad (6)$$

If these functions of τ are τ_1, τ_2, and τ_3, respectively, the differential equation becomes

$$f\,\frac{\partial^2 w}{\partial x^2} = -\frac{D}{t}\left[\tau_1\,\frac{\partial^4 w}{\partial x^4} + 2\,\tau_2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \tau_3\,\frac{\partial^4 w}{\partial y^4}\right] \qquad (7)$$

It is desirable at this point to discuss the evaluation of τ_1, τ_2, and τ_3.

When an ordinary column is loaded beyond the proportional limit, the longitudinal bending stiffness is multiplied by the factor τ, which is less than unity. Because longitudinal bending in a plate or column is the same type of action, it is reasonable to write

$$\tau_1 = \tau \tag{8}$$

Beyond the proportional limit, the term principally associated with the torsional stiffness is multiplied by τ_2. According to Bleich (reference 6), the factor τ_2 should lie between τ and unity. Since τ is always less than unity, Bleich selected

$$\tau_2 = \sqrt{\tau} \tag{9}$$

as a convenient value. Timoshenko (reference 2) also uses this value.

After analyzing the results of some 500 tests of angle columns where failure occurred by twisting, Kollbrunner (reference 7) concluded that, beyond the proportional limit, the torsional stiffness should be multiplied by the factor $(\tau + \sqrt{\tau})/2$. Thus, according to Kollbrunner,

$$\tau_2 = \frac{\tau + \sqrt{\tau}}{2} \tag{10}$$

where the values of τ are obtained from the stress-strain curve by use of the following formula:

$$\tau = \frac{4\,\dfrac{E'}{E}}{\left(1 + \sqrt{\dfrac{E'}{E}}\right)^2} \tag{11}$$

where E' is the slope of the stress-strain curve at the stress for which the value of τ is desired.

The method used by Kollbrunner to determine τ is based on the assumption that no deflection takes place until buckling occurs. Consequently, all the effects of deflection from the beginning of loading are included in his relation between τ_2 and τ. In practical engineering calculations, it is safer, as well as more expedient, to determine τ from the accepted column curve for the material. By this procedure the values of τ include effects caused by deflection from the beginning of loading. Had Kollbrunner determined the values of τ from the accepted column curve for the material, a different relation between τ_2 and τ would have been found. At a given stress, the value of τ determined from the accepted column curve for the material is smaller than the value given by equation (11). It is therefore conservative to use Kollbrunner's equation for τ_2 when the values of τ are determined from the column curve.

It seems to be common practice in the literature to assume that the transverse bending stiffness is unaffected when the longitudinal stress exceeds the proportional limit for the material. This assumption is expressed in equation form as follows:

$$\tau_3 = 1 \tag{12}$$

Although this value for τ_3 seems reasonable, it is merely an assumption. The term to which τ_2 is multiplied is about twice as important as the term to which τ_3 is multiplied. Therefore, the conservative value for τ_2 will probably compensate for any unforeseen reduction in τ_3.

Now consider equation (7). Because opposite walls of the rectangular tube are alike and symmetrical, only two equations of the type of equation (7) are required in this problem. From the solution of these equations, with proper regard for the edge conditions at the corners of the tube, a long transcendental equation for the buckled form of equilibrium is obtained. Study of this transcendental equation showed that if

$$\left.\begin{aligned} \tau_1 &= \tau \\ \tau_2 &= \sqrt{\tau} \\ \tau_3 &= 1 \end{aligned}\right\} \tag{13}$$

then

$$\eta = \sqrt{\tau} \tag{14}$$

This result is true for all values of b/h and t_b/t_h.

Further study showed that if

$$\left.\begin{aligned} \tau_1 &= \tau \\ \tau_2 &= \frac{\tau + \sqrt{\tau}}{2} \\ \tau_3 &= 1 \end{aligned}\right\} \qquad (15)$$

then

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4} \qquad (16)$$

when $b/h = 1$ and $t_b/t_h = 1$. For any other values of b/h and t_b/t_h, equation (16) gave a conservative approximation for η as indicated by the comparison of numerical values given in table III.

The values of b/h and t_b/t_h selected for the comparison made in table III were chosen to represent some of the cases in which equation (16) would be least accurate. Also the low value of $\tau = 0.1$ used as a basis for comparison was selected with the same thought in mind. For larger values of τ, the percentage error is reduced.

For comparison, the values of η given by equations (14) and (16) are plotted against τ in figure 4 in addition to the very conservative value of

$$\eta = \tau \qquad (17)$$

obtained from the condition

$$\tau_1 = \tau_2 = \tau_3 = \tau \qquad (18)$$

As a matter of interest, there is also plotted in figure 4 the relation between η and τ when $\eta = \tau_2$ and τ_2 is given by equation (10).

As a summary of this discussion, it is recognized that the proper value of the effective modulus ηE for local buckling of thin-wall rectangular tubes will depend upon tests. Careful consideration of theory and experimental data, however, indicates that it is safe to assume that η is given by equation (16) provided that τ is evaluated by use of the accepted column curve for the material.

ILLUSTRATIVE PROBLEM

It is desired to calculate the critical compressive stress at which cross-sectional distortion begins in the three 24ST aluminum alloy rectangular tubes shown in figure 5.

The critical stress is given by equation (4):

$$f_{crit} = \eta \frac{k \pi^2 E t_h^2}{12 (1-\mu^2) h^2} \tag{4}$$

If equation (4) is divided by η, the following equation is obtained:

$$\frac{f_{crit}}{\eta} = \frac{k \pi^2 E t_h^2}{12 (1-\mu^2) h^2} \tag{19}$$

The problem is to find f_{crit} when the value of f_{crit}/η has been established by equation (19).

It is assumed that the value of η is given by equation (16):

$$\eta = \frac{\tau + 3 \sqrt{\tau}}{4} \tag{16}$$

The value of τ depends upon the critical stress. Therefore, the value of η also depends upon the critical stress. Although theoretically the values of τ and hence of η can be obtained from the stress-strain curve, they are best obtained from the accepted column curve for the material.

Evaluation of τ

The equations that show the variation of τ with stress for 24ST aluminum alloy which just meets the requirements of Navy Dept. Specification 46A9a (tensile yield strength, 42,000 pounds per square inch) are given in part I of reference 8. In order to show how similar equations can be derived for any other material, these equations will be derived from the column formulas given in reference 8 for 24ST aluminum alloy.

The accepted column formulas for 24ST aluminum alloy are given by equations (8) and (9) of reference 8. These equations are, respectively:

For $41{,}200 > f_{crit} > 19{,}600$ lb./sq. in.

$$f_{crit} = 43{,}700\ (1 - 0.00752\ L/\rho) \qquad (20)$$

For $f_{crit} < 19{,}600$ lb./sq. in.

$$f_{crit} = \frac{105200000}{\left(\frac{L}{\rho}\right)^2} \qquad (21)$$

For the same member, the critical stress given by the accepted column curve must be equal to the critical stress given by the Euler formula with an effective modulus $\bar{E} = \tau E$ substituted for Young's modulus E; or

$$f_{crit} = \frac{\pi^2\ \tau\ E}{\left(\frac{L}{\rho}\right)^2} = \frac{\tau\ 105200000}{\left(\frac{L}{\rho}\right)^2} \qquad (22)$$

Now, if equations (20) and (21) are solved for L/ρ, the following expressions are obtained:

For $41{,}200 > f_{crit} > 19{,}600$ lb./sq. in.

$$\frac{L}{\rho} = \frac{43700 - f_{crit}}{328.6} \qquad (23)$$

For $f_{crit} < 19{,}600$ lb./sq. in.

$$\frac{L}{\rho} = \sqrt{\frac{105200000}{f_{crit}}} \qquad (24)$$

Substitution of these values of L/ρ in equation (22) and solving for τ gives

For $41{,}200 > f_{crit} > 19{,}600$ lb./sq. in.

$$\tau = \frac{f_{crit}}{8925}\left(1.224 - \frac{f_{crit}}{35700}\right)^2 \qquad (25)$$

For $f_{crit} < 19{,}600$ lb./sq. in.

$$\tau = 1 \qquad (26)$$

Equations (25) and (26) are the same as equations (15) and (16) of reference (8).

Evaluation of the Critical Stress for Cross-Sectional Distortion

By the use of equations (25) and (26), the value of τ can be established for assumed values of f_{crit}. The values of τ obtained can then be substituted in equation (16) to obtain the corresponding values of η. If the assumed values of f_{crit} are divided by the corresponding values of η, a curve of f_{crit} against f_{crit}/η can be plotted. The critical stress at which cross-sectional distortion begins in the three 24ST aluminum-alloy rectangular tubes shown in figure 5 can then be calculated by the use of equation (19) and the curve of f_{crit} against f_{crit}/η.

The solid curve in figure 6 shows the relation between f_{crit} and f_{crit}/η for 24ST aluminum alloy calculated in the manner outlined. The three additional curves in figure 6 were obtained by the equation for η noted on each curve. The calculated values used to establish the curves of figure 6 are given in table IV.

The critical stress for cross-sectional distortion of the tubes in figure 5 is obtained as follows:

Tube A

$$\frac{t_b}{t_h} = \frac{0.084}{0.084} = 1.0$$

$$\frac{b}{h} = \frac{0.92}{1.84} = 0.5$$

$$k = 5.16 \text{ (read from fig. 3)}$$

$$E = 10.66 \times 10^6 \text{ lb./sq. in.}$$

$$\mu = 0.3$$

From equation (19)

$$\frac{f_{crit}}{\eta} = \frac{5.16 \times \pi^2 \times 10.66 \times 10^6 \times (0.084)^2}{12\ (1-0.3^2) \times (1.84)^2} = 103{,}600 \text{ lb./sq. in.}$$

From the solid curve of figure 6

$$f_{crit} = 36{,}400 \text{ lb./sq. in.}$$

Tube B

$$\frac{t_b}{t_h} = \frac{0.042}{0.084} = 0.5$$

$$\frac{b}{h} = \frac{0.92}{1.84} = 0.5$$

$$k = 4.11 \quad \text{(read from fig. 3)}$$

$$E = 10.66 \times 10^6 \text{ lb./sq. in.}$$

$$\mu = 0.3$$

From equation (19)

$$\frac{f_{crit}}{\eta} = \frac{4.11 \times \pi^2 \times 10.66 \times 10^6 \times (0.084)^2}{12\ (1 - 0.3^2) \times (1.84)^2} = 82{,}530 \text{ lb./sq. in.}$$

From the solid curve of figure 6

$$f_{crit} = 34{,}900 \text{ lb./sq. in.}$$

Tube C

$$\frac{t_b}{t_h} = \frac{0.042}{0.021} = 2.0$$

$$\frac{b}{h} = \frac{0.92}{1.84} = 0.5$$

$$k = 6.59 \quad \text{(read from fig. 3)}$$

$$E = 10.66 \times 10^6 \text{ lb./sq. in.}$$

$$\mu = 0.3$$

From equation (19)

$$\frac{f_{crit}}{\eta} = \frac{6.59 \times \pi^2 \times 10.66 \times 10^6 \times (0.021)^2}{12\ (1 - 0.3^2) \times (1.84)^2} = 8,270 \text{ lb./sq. in.}$$

Because $f_{crit}/\eta < 19,600$ lb./sq. in., it follows from figure 6 that

$$f_{crit} = \frac{f_{crit}}{\eta} = 8,270 \text{ lb./sq. in.}$$

Had it been assumed that $\eta = \sqrt{\tau}$, the value of f_{crit} for tube A would have been read from the curve for $\eta = \sqrt{\tau}$ in figure 6 and would have been 37,400 pounds per square inch instead of 36,400. The critical stress is thus raised only about 3 percent by using the least conservative value of η considered herein. If the very conservative value of $\eta = \tau$ is used, the critical stress for tube A is read from the curve for $\eta = \tau$ in figure 6, which gives $f_{crit} = 33,200$ pounds per square inch. The critical stress is thus lowered about 9 percent by using the most conservative value of η.

The ultimate compressive strength of a thin-wall tube of rectangular section will, in general, be higher than the load at which cross-sectional distortion begins. At stresses approaching the yield point for the material, the critical load and the ultimate load approach the same value. No attempt has been made in this paper to discuss the ultimate strength of a thin-wall tube of rectangular section; the solution for the critical load logically precedes the solution for the ultimate load.

CONCLUSIONS

1. The critical compressive stress at which cross-sectional distortion occurs in a thin-wall rectangular tube

symmetrical about its two principal axes is given by the equation

$$f_{crit} = \eta \frac{k \pi^2 E t_h^2}{12 (1 - \mu^2) h^2}$$

whore

E and μ are Young's modulus and Poisson's ratio, respectively, for the material.

h and t_h, the width and the thickness, respectively, of the wider walls.

k, a coefficient dependent upon the relative dimensions of the tube, minimum values of which may be obtained from figure 3.

η, a factor taken so that ηE gives the effective modulus of the walls at stresses beyond the elastic range.

2. The value of the effective modulus ηE for local buckling of thin-wall rectangular tubes will depend upon tests. In the absence of such tests, however, it is reasonable to assume that η is a function of τ, where τE is the effective modulus of an ordinary column at stresses beyond the elastic range. A careful study of the theory and such experimental data as are available indicates that <u>it is safe to assume</u> that η is given by the equation

$$\eta = \frac{\tau + 3 \sqrt{\tau}}{4}$$

provided that τ is evaluated by use of the accepted column curve for the material.

It is important to mention here that, when η is considered to be a function of τ, the equation for η will depend upon the manner of the evaluation of τ. If τ is determined from the stress-strain curve on the assumption that no deflection takes place until the critical stress is reached, the effect of deflections from the beginning of loading must be separately considered. If τ is determined, however, from the accepted column curve for the material in the manner outlined in the illustrative problem, part, if not all, of this effect is automatically considered.

3. Because η is a function of τ, which is a function of the critical stress, a curve of f_{crit} against f_{crit}/η should first be plotted for the material by means of the method of calculation outlined in the illustrative problem. Then, in a given problem, f_{crit}/η can be computed from the formula

$$\frac{f_{crit}}{\eta} = \frac{k \pi^2 E t_h^2}{12 (1 - \mu^2) h^2}$$

and the critical stress can be read from this curve.

Langley Memorial Aeronautical Laboratory,
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TABLE I

Calculated Minimum Values of k

by the Energy Solution

	k		
$\frac{b}{h}$ \ t_b/t_h	0.5	1	2
0	7.01	7.01	7.01
.050	5.13	6.45	--
.075	4.88	--	--
.100	4.72	6.09	6.85
.125	4.62	--	--
.200	4.43	5.68	6.73
.300	4.31	5.45	6.65
.400	4.22	5.29	6.61
.500	4.11	5.16	6.59
.525	4.08	--	--
.550	4.04	--	--
.575	4.00	--	--
.590	3.97	--	--
.600	3.95	5.03	6.57
.610	3.92	--	--
.625	3.89	--	--
.650	3.81	--	--
.675	3.64	--	--
.700	3.38	4.87	6.57
.800	2.58	4.66	6.57
.900	2.03	4.37	6.57
1.000	1.64	4.00	6.58

TABLE II

Comparison of Values of k Computed

by the Energy Solution and

the Exact Solution

$\frac{t_b}{t_h}$	$\frac{b}{h}$	k (energy)	k (exact)	Error (percent)
0.5	0	7.0074	6.9707	0.524
	.3	4.3066	4.3064	.005
	.6	3.9469	3.9469	0
	.7	3.3785	3.3485	.888
	1.0	1.6441	1.6377	.391
1.0	0	7.0074	6.9707	0.524
	.3	5.4471	5.4395	.140
	.7	4.8697	4.8672	.051
	1.0	4.0000	4.0000	0
2.0	0	7.0074	6.9707	0.524
	.3	6.6513	6.6245	.403
	.7	6.5682	6.5453	.349
	1.0	6.5764	6.5507	.391

TABLE III

Comparison of Values of η Given by the Equation

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4}$$

with the Exact Values Found by

Solving the Transcendental Equation

$\frac{b}{h}$	$\frac{t_b}{t_h}$	$\eta = \frac{\tau+3\sqrt{\tau}}{4}$	η exact value	Error (percent)
0	0.5	0.262	0.278	5.76
.6		.262	.273	4.03
.7		.262	.275	4.73
0	1.0	0.262	0.278	5.76
.7		.262	.266	1.50
1.0		.262	.262	0
0	2.0	0.262	0.278	5.76

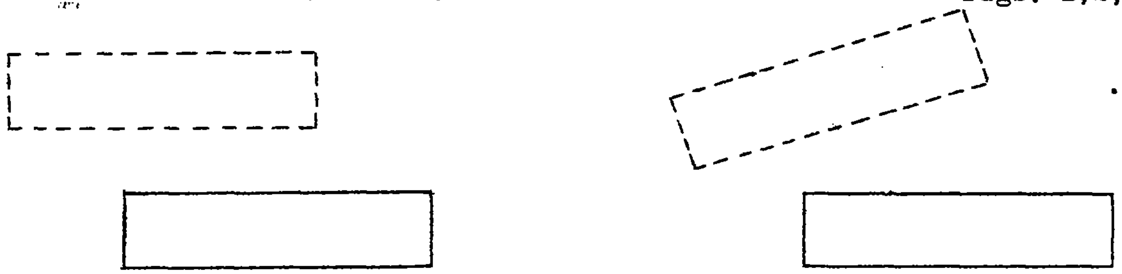
In all calculations for this table, $\tau_1 = \tau$, $\tau_2 = \frac{\tau + \sqrt{\tau}}{2}$, $\tau_3 = 1$, and $\tau = 0.1$.

TABLE IV

Values Used to Establish Curves in Figure 6

(All values in pounds per square inch)

η / f_{crit}	$\frac{f_{crit}}{\eta}$			
	τ	$\frac{\tau + \sqrt{\tau}}{2}$	$\frac{\tau + 3\sqrt{\tau}}{4}$	$\sqrt{\tau}$
20,000	20,260	20,190	20,160	20,130
22,000	24,160	23,590	23,320	23,050
24,000	29,320	27,860	27,170	26,530
26,000	36,330	33,290	31,960	30,730
28,000	46,170	40,430	38,060	35,960
30,000	60,640	50,080	46,070	42,660
32,000	83,180	63,680	57,010	51,600
34,000	120,950	83,810	72,670	64,130
36,000	192,000	116,020	96,880	83,140
38,000	350,230	173,520	138,580	115,360
40,000	831,770	299,180	226,630	182,400
41,200	1,827,000	476,960	348,270	274,300



(a) Translated (b) Translated and rotated

Figure 1.- Displacements of the cross section in primary, or general, failure of a column.

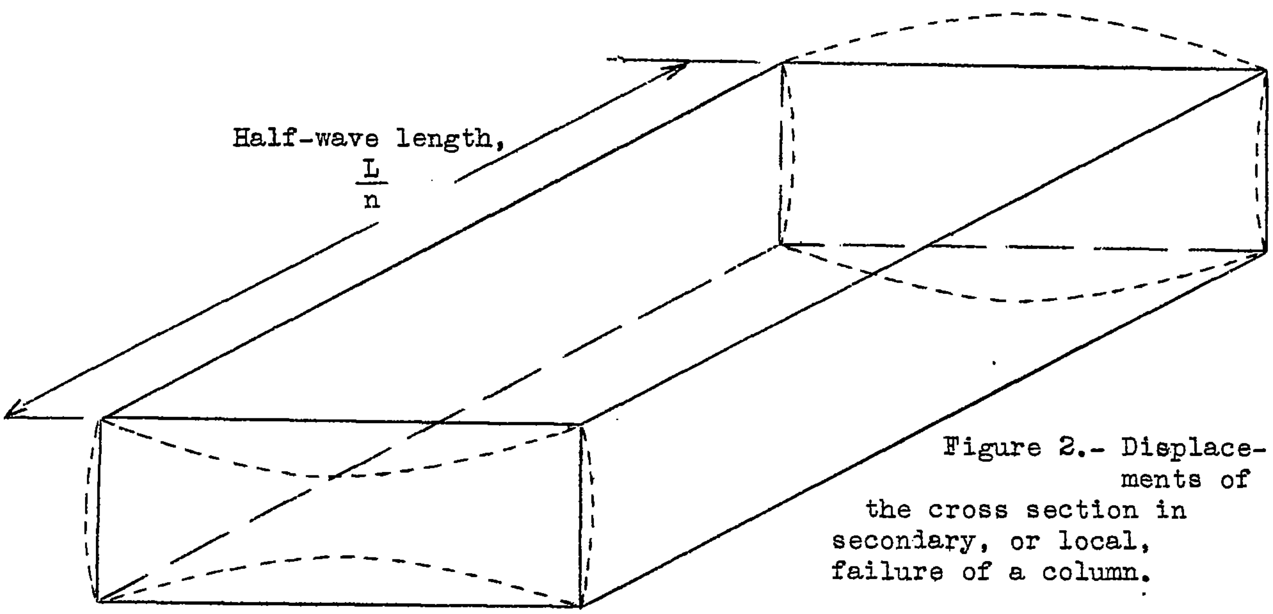
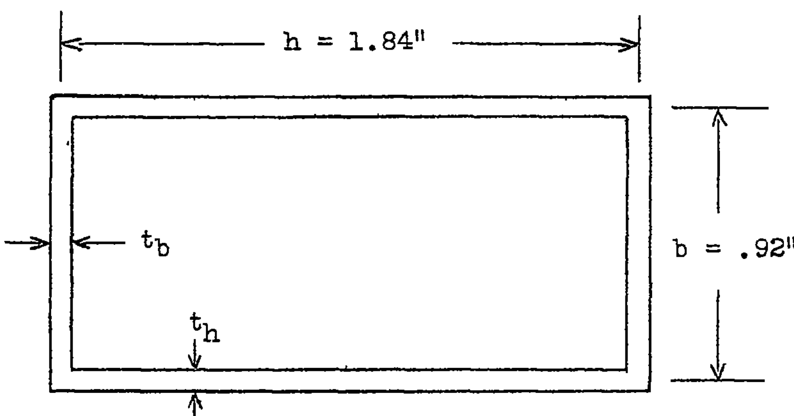


Figure 2.- Displacements of the cross section in secondary, or local, failure of a column.



	t_b(in.)	t_h(in.)
Tube A	0.084	0.084
" B	.042	.084
" C	.042	.021

Figure 5.- Tubes used in illustrative problem.

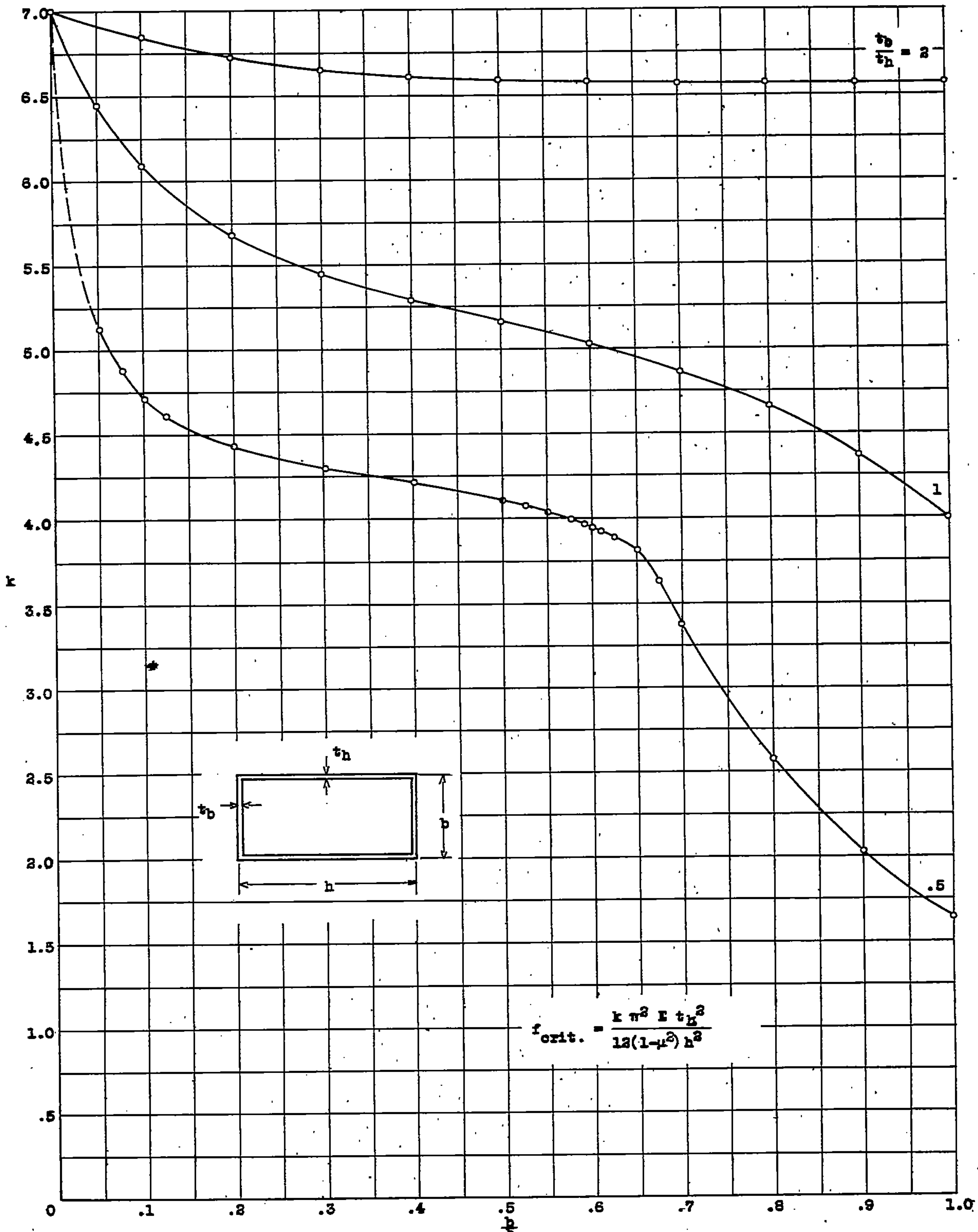


Figure 3.- Minimum theoretical values of k for a symmetrical rectangular tube.

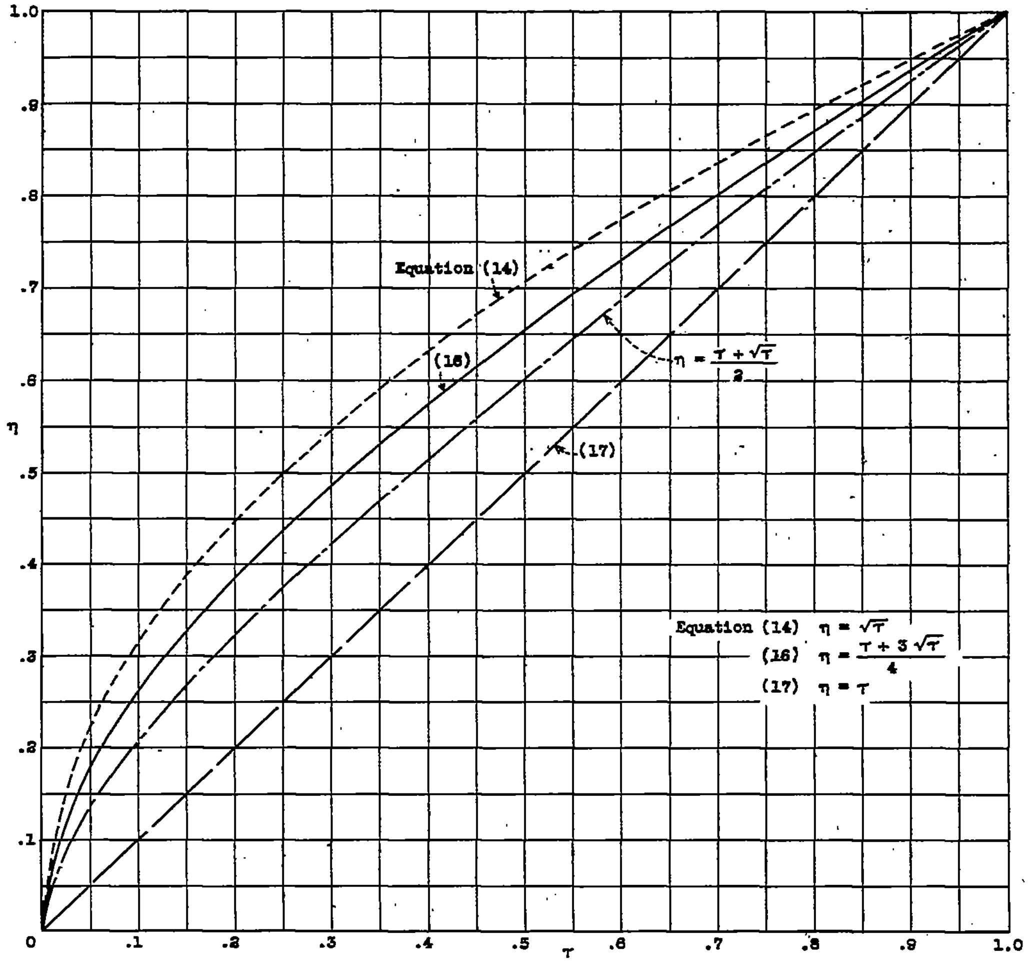


Figure 4.- Variation of η with τ as given by different equations.

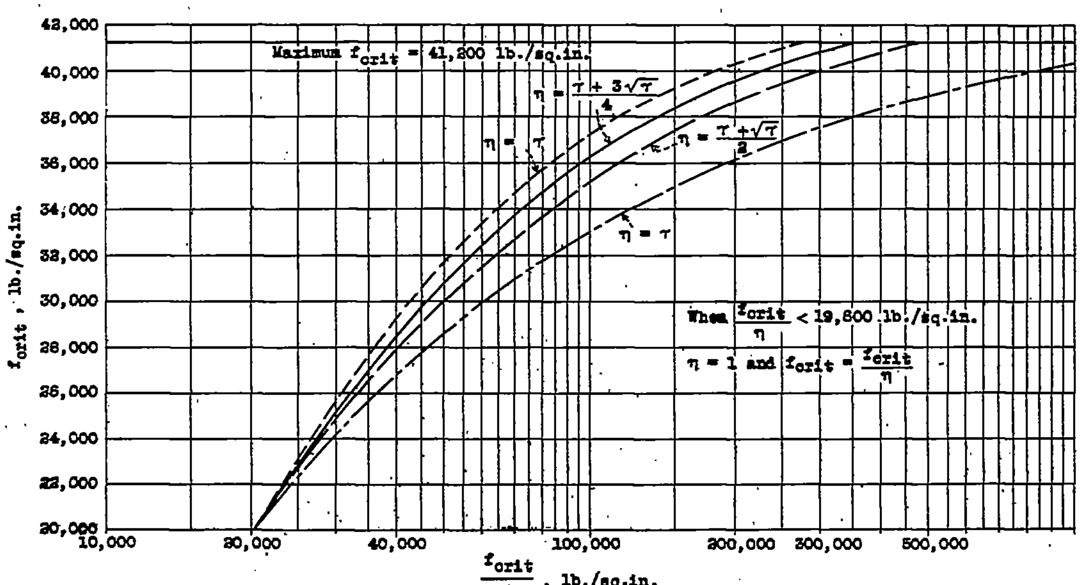


Figure 6.- Variation of f_{crit} with $\frac{f_{crit}}{\eta}$ for 24 ST aluminum-alloy rectangular tubes.